Physics of memories

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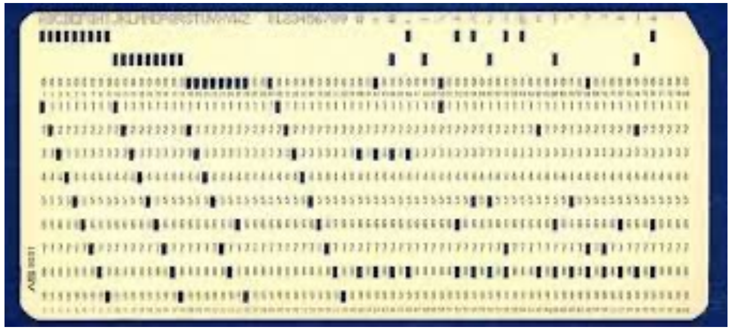
The big picture

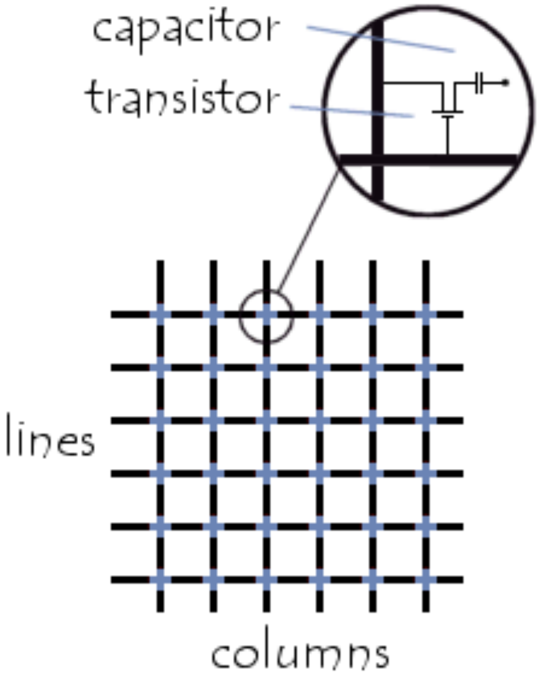
All automatic digital computing activities are made with elementary devices that we call combinational binary switches.
These devices can be combined together to make logic gates and memories, that are the basic components of modern computers, according to the von Neumann architecture.
We have shown that all the computing activity with these devices can be performed without spending any energy.

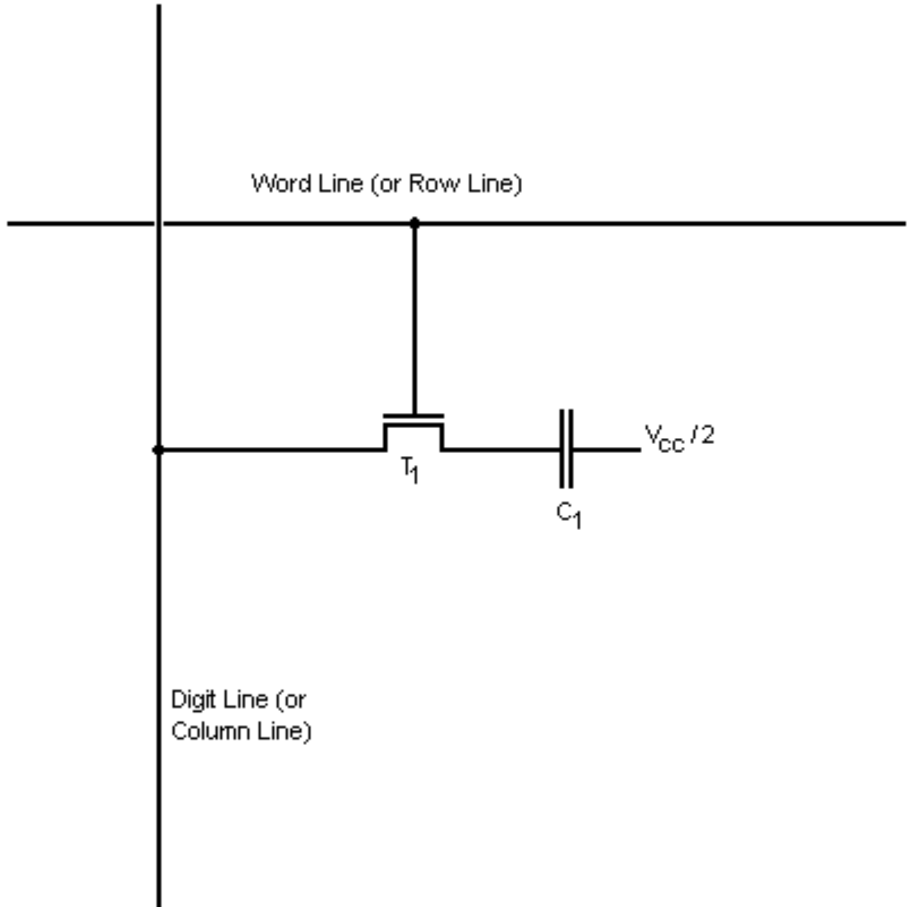
What about memories?



A common phenomenon: memory degradation

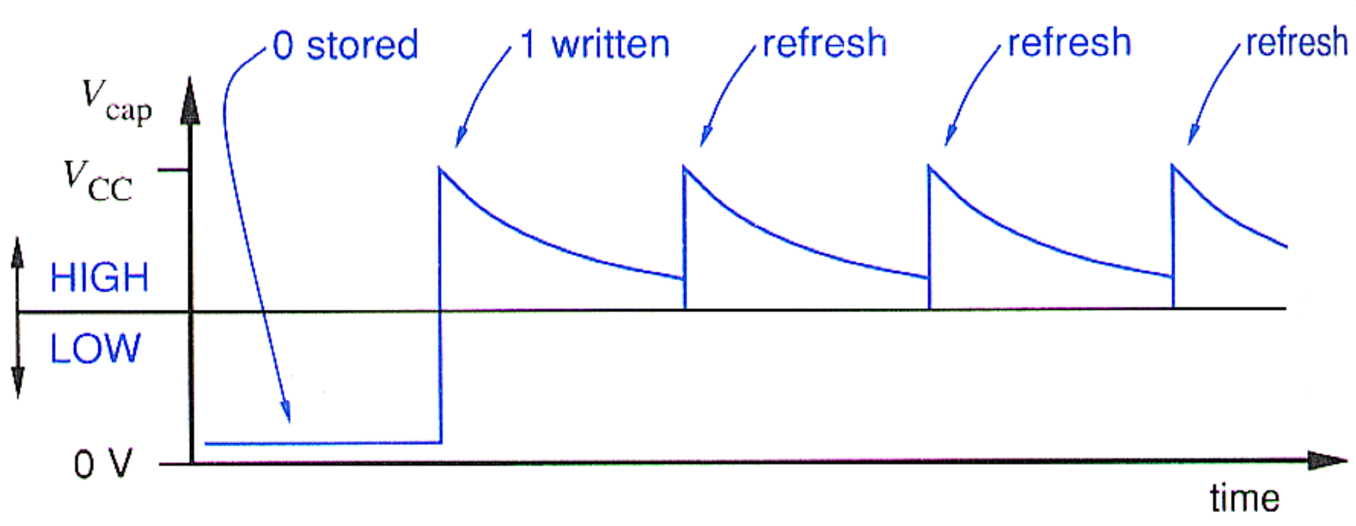
Transistor based memory





DRAM

In order to counterbalance the memory degradation, a periodic refresh operation is performed



If no refresh operation is performed the memory is lost on average after a time τ_K

The refresh operation is performed periodically with period t_R

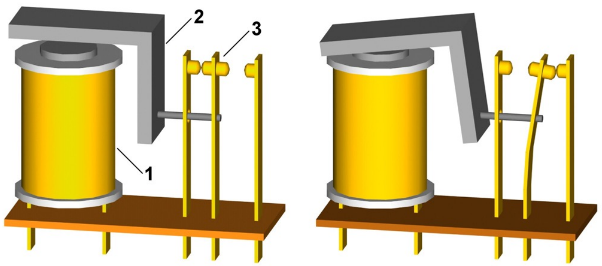
The refresh operation last for a time t_p

$$t_p << t_R << \tau_K$$

Nano seconds (t_p), Micro seconds (t_R), seconds (τ_K)

Binary switches

Combinational switches can be easily employed do computation.
Sequential switches can be easily employed to store information.



Conbinational:
in the absence of any external force, under equilibrium conditions, they are in the state S_0. When an external force F_{01} is applied, they switch to the state S_1 and remain in that state as long as the force is present. Once the force is removed they go back to the state S_0.

Sequential:
They can be changed from S_0 to S_1 by applying an external force F_{01}. Once they are in the state S_1 they remain in this state even when the force is removed. They go from S_1 to S_0 by applying a new force F_{10}. Once they are in S_0 they remain in this state even when the force is removed.

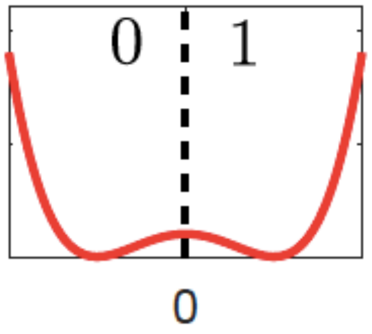
Plan of the work

Assumed that the refresh operation has an energetic cost **Q**, we are interested in determining the fundamental energy limits to preserve a given bit for a given time **t**, with a probability of failure not larger than $\mathbf{P_E}$, while executing the refresh procedure with periodicity $\mathbf{t_R}$.

1. introduce a simple physical model for the 1-bit memory
2. Compute $\mathbf{t_R}$ for a given set of $\mathbf{P_E}$ and **t**
3. Perform an experiment to determine the minimum energy required
4. Elaborate considerations

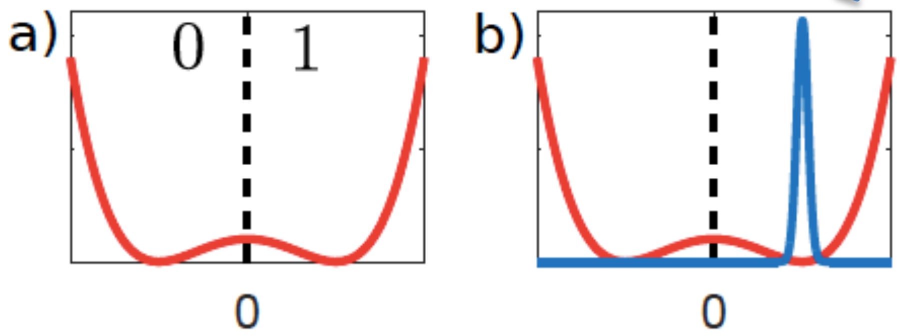
1 1-bit memory

a)



1 1-bit memory

Initial probability density p(x,t)



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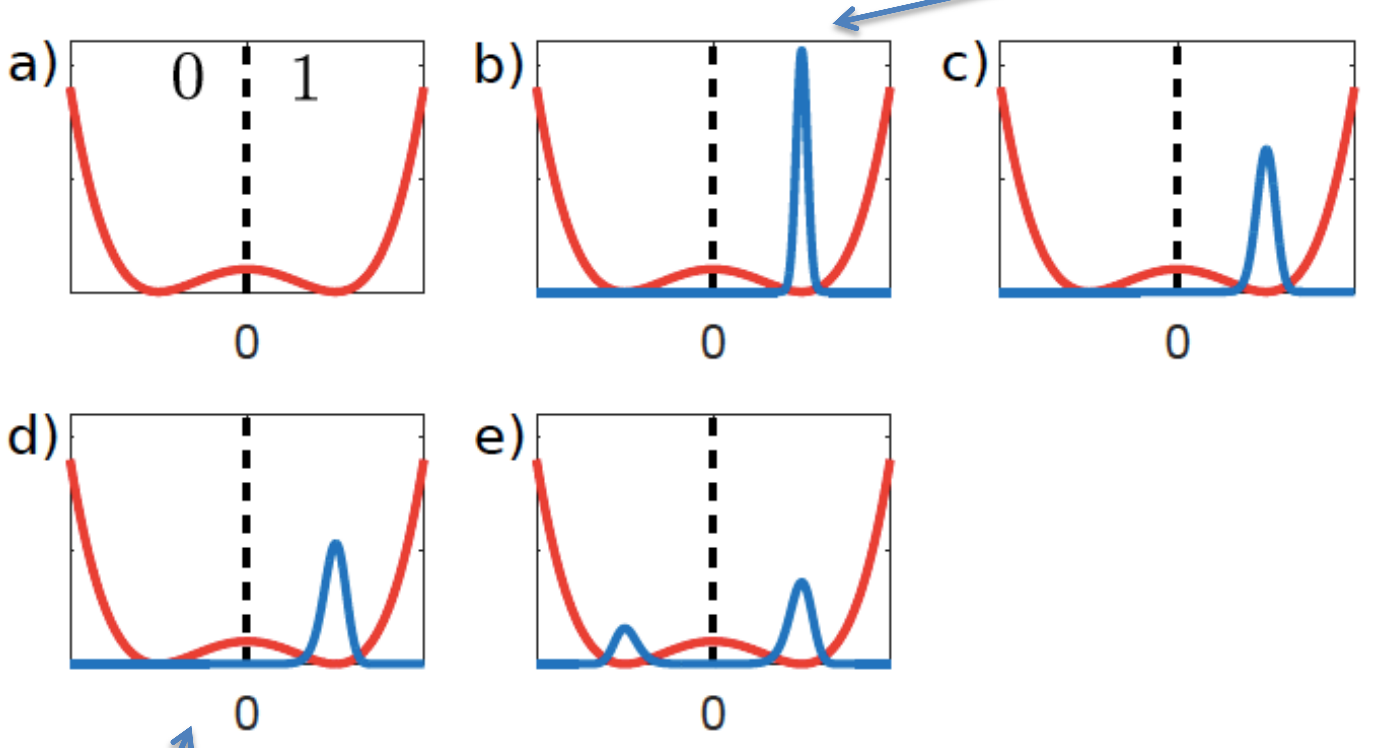

Relaxation within one well
With time scale t_w

Relaxation toward global
Equilibrium with time scale τ_K

1 1-bit memory

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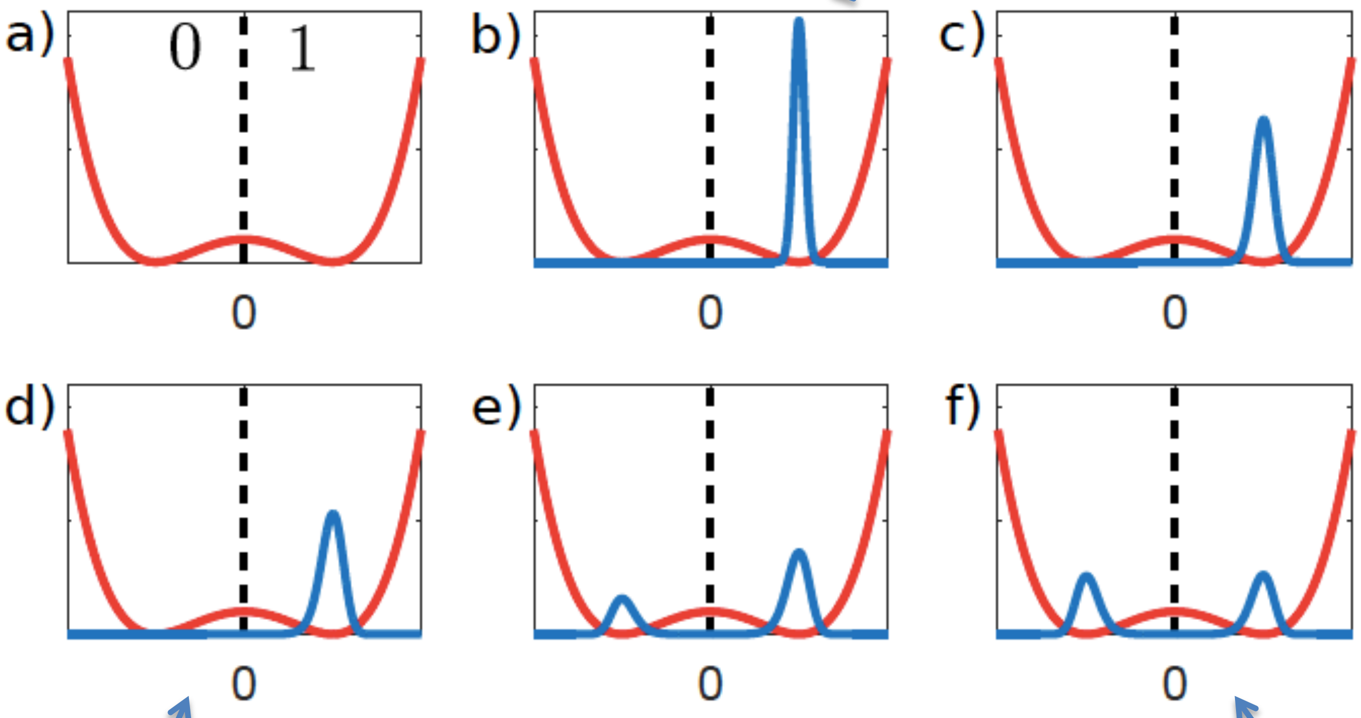


Relaxation within one well
With time scale t_w

Relaxation toward global
Equilibrium with time scale τ_K

Memory lost

2 Compute t_R for a given set of P_E and t

In this framework if we indicate with $P_0(t) = \int_{-\infty}^{0} p(x,t)\mathrm{d}x$

the probability to be in the wrong well (bit 0 instead of bit 1), we have:

$$P_E = 1 - \left[1 - P_0\left(t_R\right)\right]^{\frac{t}{t_R}}$$

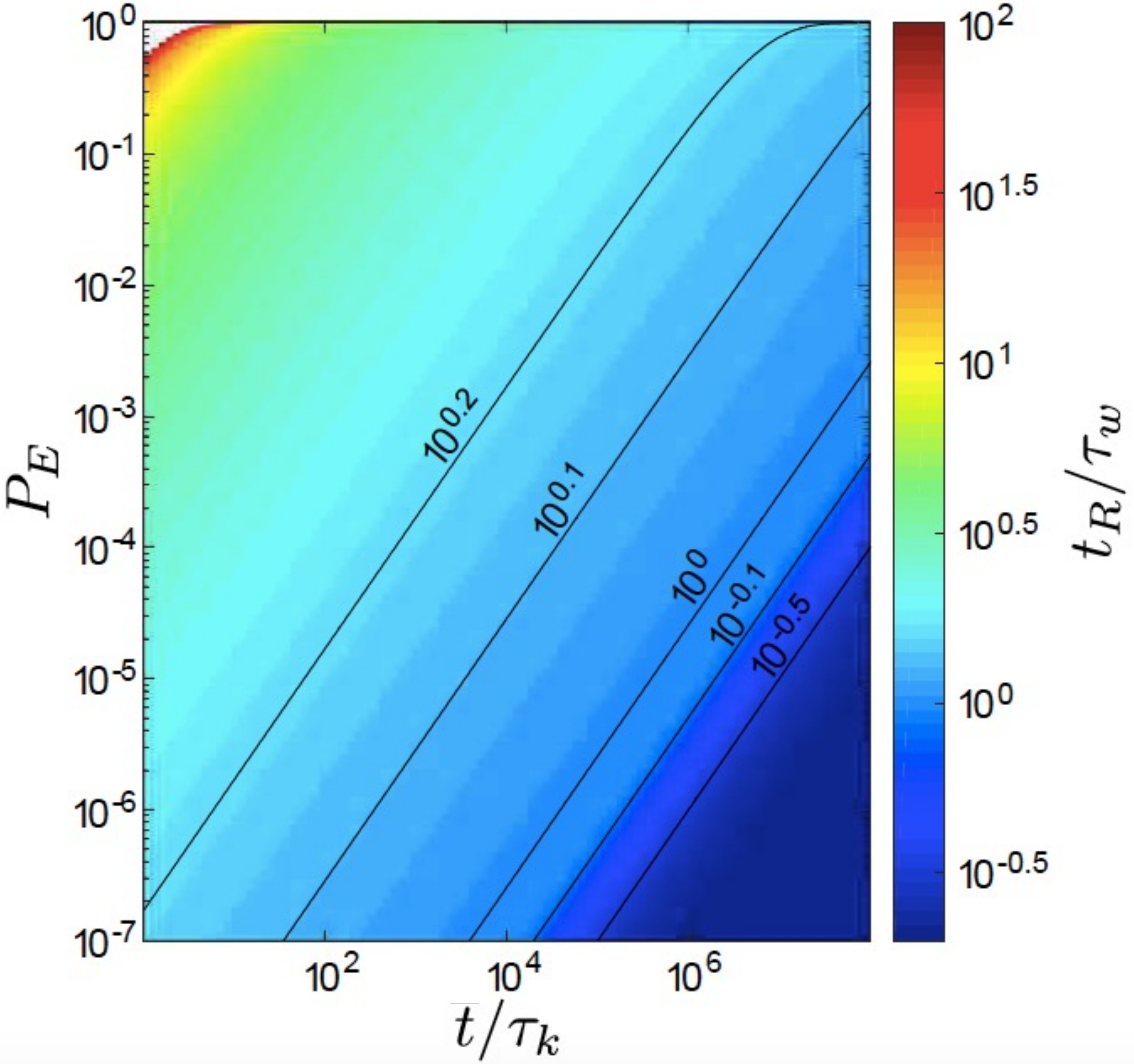
After $N = t/t_R$ refresh cycles

The density function p(x, t) is described via the dimensionless Fokker-Plank equation

$$\frac{\partial}{\partial t}\, p(x,t) = \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial x}\, p(x,t)\right) + \frac{k_B T}{\Delta U}\frac{\partial^2}{\partial x^2}\, p(x,t)$$

In order to compute this quantity we assume a bistable Duffing potential U(x).

2 Compute t_R for a given set of P_E and t



3 Determine the minimum energy for keeping the memory

We now consider the energy cost of a single refresh operation.
Based on our model, the refresh operation consists in bringing the p(x,t) back to its initial condition:

$$p(x,t_R) \rightarrow p(x,0)$$

We assume that the motion inside one well can be approximated by the harmonic oscillator dynamics. This is reasonable while $t_R << \tau_K$.

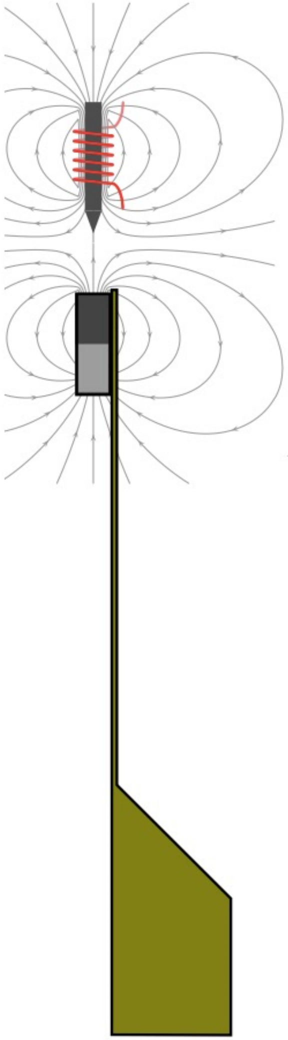
The resulting probability density function is a sum of two Gaussian peaks centred around the minima of **U**(**x**), each one with the same standard deviation σ

The refresh operation amounts to change $\sigma_f = \sigma(t_R)$ into $\sigma_i = \sigma(0)$

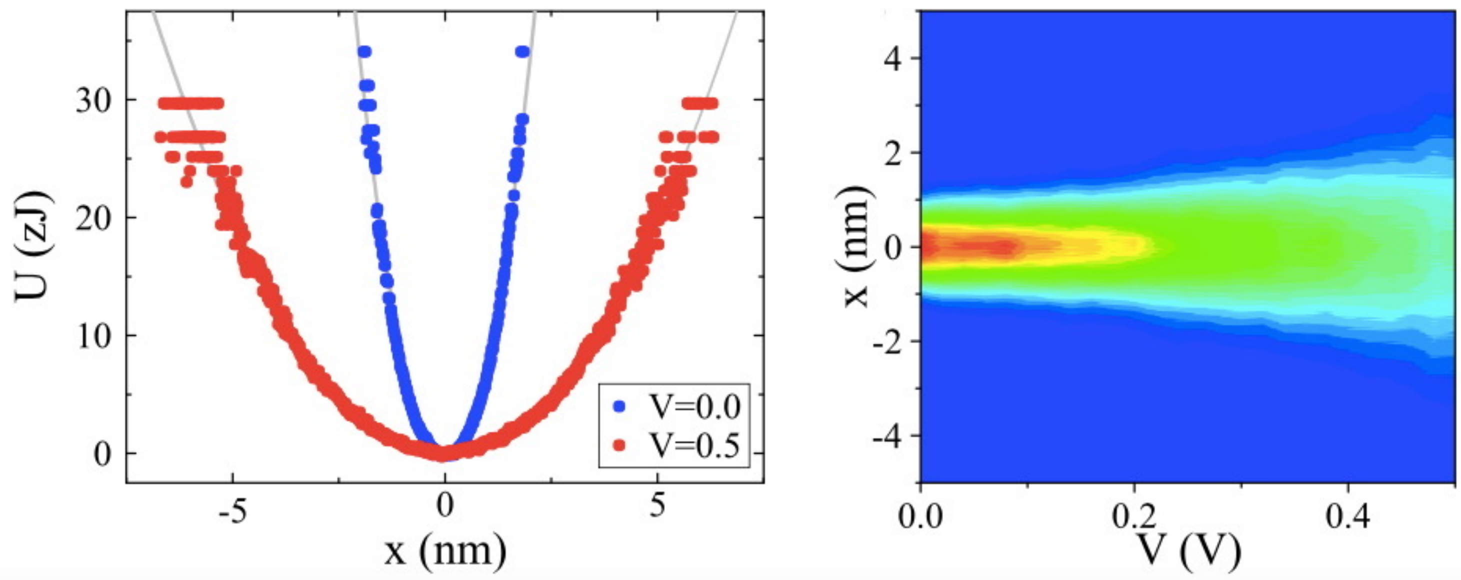
with

$$\sigma(t_R) = \sqrt{\sigma_w^2 + \exp\left(-\frac{t_R}{\tau_w}\right)(\sigma_i^2 - \sigma_w^2)}$$

3 Determine the minimum energy for keeping the memory



To measure the minimum energy required for the refresh, i.e. to "squeeze" the density function inside an harmonic well, we perform an experiment with a micro-mechanical V-shaped cantilever where the relevant observable x is the tip position.



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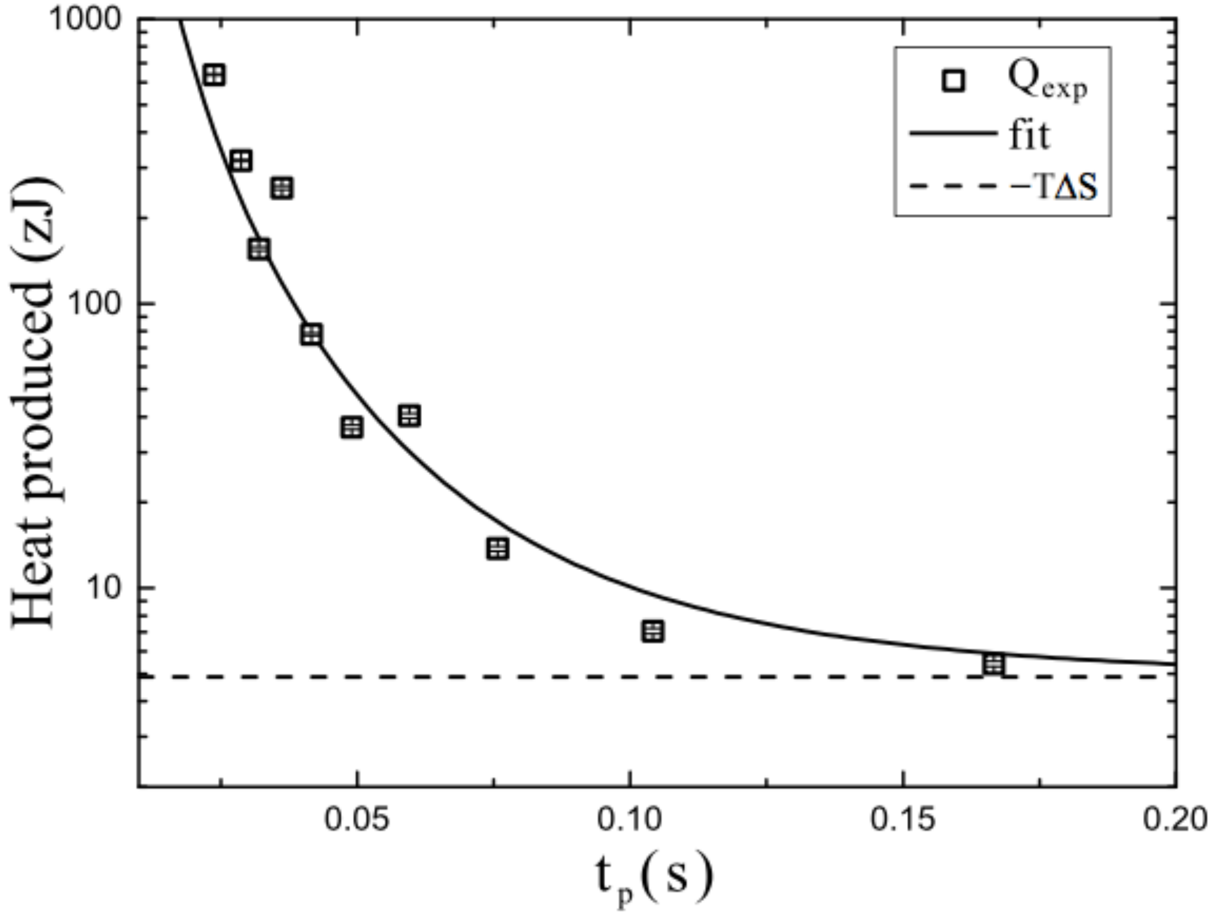
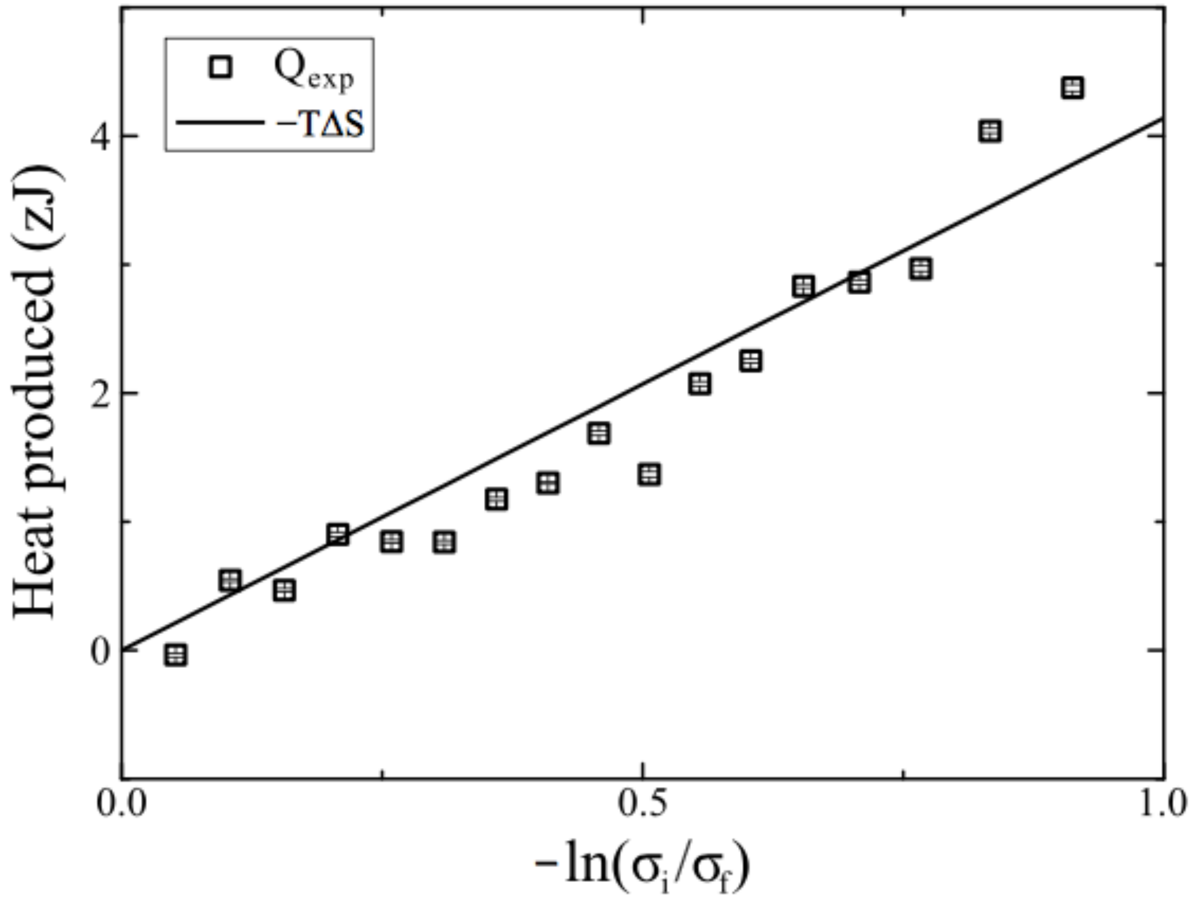


T ΔS

3 Determine the minimum energy for keeping the memory



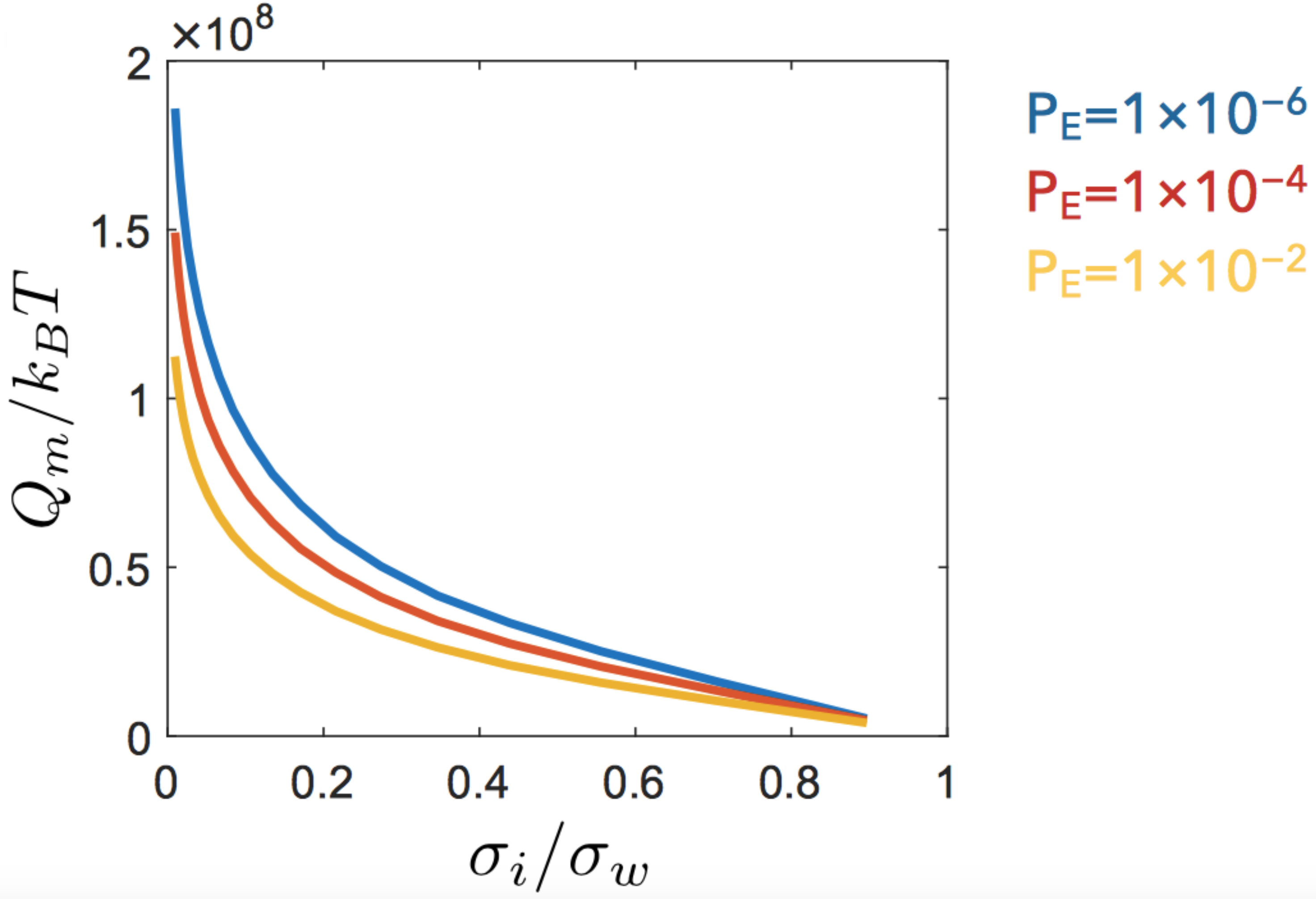
With $\Delta S = k_B \ln\left(\frac{\sigma_i}{\sigma_f}\right)$

3 Determine the minimum energy for keeping the memory

MINIMUM ENERGY REQUIRED TO PRESERVE A MEMORY OVER A FIXED TIME WITH A GIVEN ERROR PROBABILITY

$$Q_m = -NT\Delta S = \frac{t}{t_R} k_B T \ln \left(\frac{\sqrt{(\sigma_w^2 + e^{-\frac{t_R}{\tau_w}} (\sigma_i^2 - \sigma_w^2)}}{\sigma_i} \right)$$

3 Determine the minimum energy for keeping the memory



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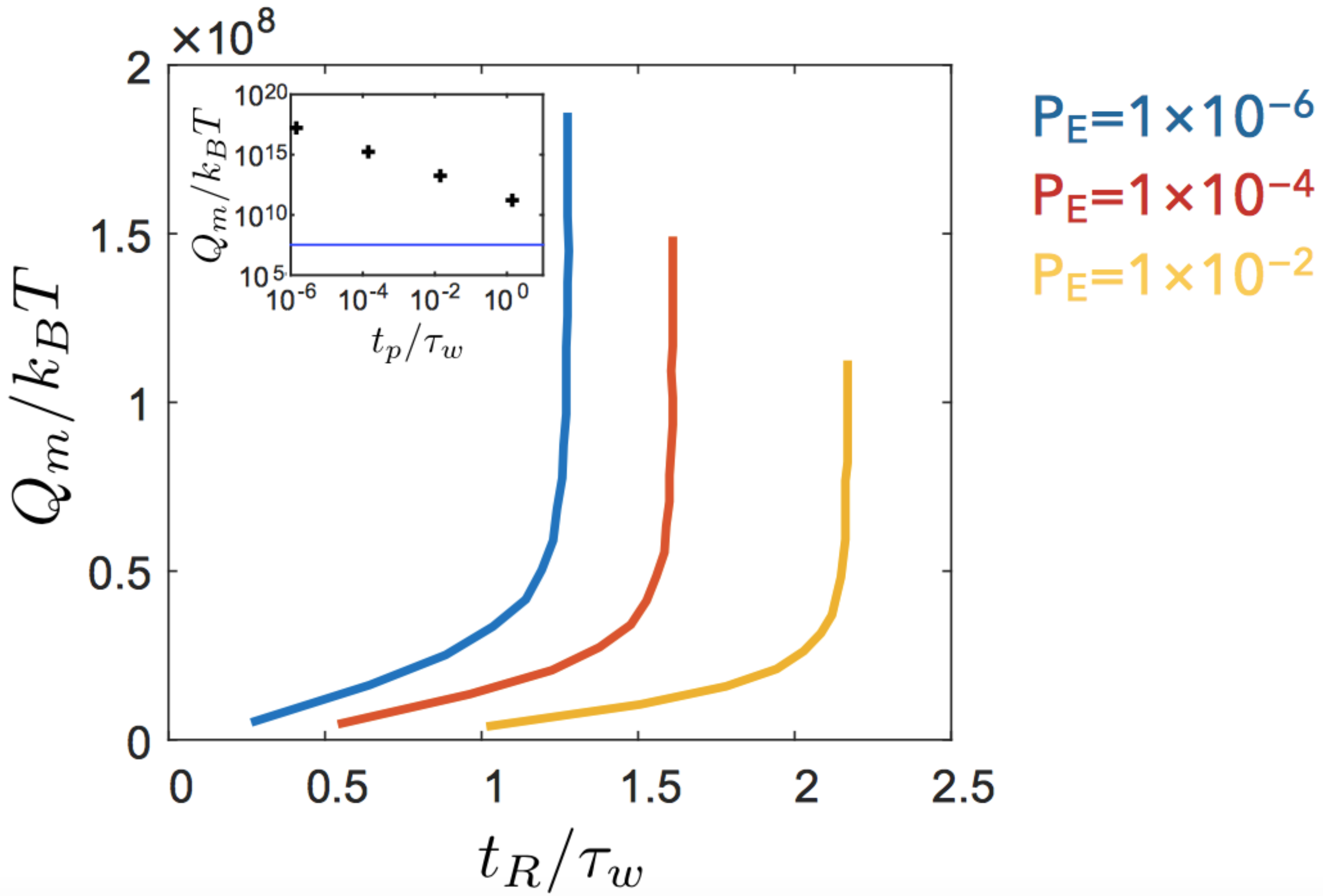

4 Considerations

The good news!

We can preserve a memory for a given time with a given error probability while spending an arbitrarily little amount of energy.

This is accomplished if the refresh procedure is performed arbitrarily often or arbitrarily close to thermal equilibrium.

In both cases, what is important is that there is a negligible entropy variation!

4 Considerations

The bad news!

If we consider the relation: $$P_E = 1 - \left[1 - P_0\left(t_R\right)\right]^{\frac{t}{t_R}}$$

we have: $$t = t_R \ln(1 - P_E) / \ln(1 - P_0)$$

Once we set P_E and select a finite t_R, we can make **t** as large as we want by properly selecting P_0 small enough.

However P_0 cannot be made arbitrarily small because we cannot make $\sigma_i = 0$. In fact this is limited by the Heisemberg Indetermination principle that prevent the arbitrary confinement of the probability density.

In the best scenario we have: $$\sigma_x \sigma_p = \frac{\hbar}{2}$$

with $\sigma_p = m\sqrt{\langle v^2\rangle - \langle v\rangle^2} = \sqrt{mk_B T}$ we have $\sigma_x = \frac{\hbar}{2\sqrt{mk_B T}}$

Thus it exists a σ_{MIN} and $\sigma_i \geq \sigma_{iMin} = \frac{\hbar}{2\sqrt{mk_B T}}$

The existence of a σ_{MIN} implies that, even at **t** = 0, the probability of error P_0 is greater than zero.

4 Considerations

The bad news! A memory cannot last forever

Due to the existence of a finite minimum P_0

$$t = t_R \ln(1 - P_E)/\ln(1 - P_0)$$

for any P_E we select we have an associate maximum for the memory duration **t**

Example: If we assume the distance between the two wells $\mathbf{x}_m$ = 1 nm and a refresh period $\mathbf{t}_R$ = 6.6 10^{-3} s, we have that the minimum σ_i = 9. 6 10**–20** m.

For $\boldsymbol{P_E}$ = 1 10^{-6} then the maximum value for **t** is approximately 2 years.
For $\boldsymbol{P_E}$ = 1 10^{-4} then the maximum time **t** is approximately 200 years.

4 Considerations

Take home message

You can preserve your memory only for a limited amount of time.
Within this limit, if you do things carefully enough, you do not need to spend any energy.

More on:

The cost of remembering
Davide Chiuchiù, Miquel López-Suárez, Igor Neri, Maria Cristina Diamantini, Luca Gammaitoni.
arXiv:1703.05544, 2017